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ROBUST SLIDING MODE CONTROL FOR ELECTRICAL DRIVES

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ABSTRACT

In this paper, the same Sliding Mode Controller with time-varying switching surface is applied to obtain a high precision positioning of the drive whatever the configuration of the linked electrical machine (dc motor, three-phase induction machine and six-phase induction machine). The proposed control scheme is insensitive to mechanical configuration variations and also fault tolerant (six-phase induction machine). The efficiency of the method is validated experimentally using a test bed built up in our laboratory.

Key words: Sliding mode control; Variable structure control; Electrical drive.


1. INTRODUCTION

In recent years, intensive interest have been developed on the theory and applications of Sliding Mode Control (SMC), named also Variable Structure Control (VSC), for electrical machines such as dc motor [1], synchronous motor [2], induction motor [3], stepping motor [4] or piezoelectric machines [5] since it is a robust control strategy that is simple and easy to implement. Indeed, compared to classical advanced control theories such as adaptive or optimal control strategies, VSC requires only few computational operations and is also robust between external disturbances and plant parameters variations.

In robotics or in numerical control of tools machines fields, it has been proved that, using a time-varying switching surface, a high precision positioning of a dc [6] or an ac [7] machine can be obtained whatever the mechanical configuration (variations of load torque and inertia changes).

Now that insensitivity to the mechanical configuration for electrical drives has been experimentally demonstrated, another goal is to increase the reliability of the drive in electrical fault tolerant modes. For this, dc machines and three-phase induction machines are no more suitable and multiphase machines have been introduced [8]. Indeed, by using more than three
phases, the electrical machine is still running when several phases are lost in opened circuit mode since three of them are remaining.

One particular multiphase machine configuration that is often used for reliable positioning tasks in industrial applications is the symmetrical six-phase induction machine (6PIM) [9]. As said just before, a 6PIM is still running when up to three phases are lost. Nevertheless, large torque oscillations and weak tracking capability may happen in faulted conditions since there is an unbalanced structure between the stator and the rotor while the parameters of the machine model are widely changing. To cope with this problem, one solution is to associate a robust and simple control algorithm, such as VSC, with the field-oriented control (FOC) scheme [10].

In this way, we will introduce in this paper a SMC scheme with time-varying switching surface that is efficient in fault tolerant mode, insensitive to mechanical configuration changes and suitable whatever the machine. Indeed, the same algorithm will be first applied to a classical dc drive, a classical three-phase induction machine (that are not fault tolerant due to the machine configuration) and then to a six-phase induction machine when up to two stator phases are lost. For these three kinds of machines, experimental validations will be shown.

The first part of this paper will be dedicated to SMC strategy and especially to SMC scheme with time-varying switching surface. Then the second part will be devoted to the application of the proposed strategy for high precision positioning of a dc motor and three-phase induction machine (3PIM) when mechanical load variations occur. Experimental results will be introduced. Finally, still the same SMC algorithm will be implemented to a six-phase induction machine 6PIM in order to obtain a high precision positioning of the drive in fault tolerant mode and when mechanical configuration variations occur.

2. SLIDING MODE CONTROL STRATEGY

2.1. Concept

SMC theory, that was originally elaborated in the past Soviet Union in such a way to stabilize systems modeled by high order differential equations [11], is based on the idea that the topology of the system is varied intentionally so that the system response follows a predefined trajectory. Also, there are different ways to define the parameters of each structure and to define the switching logic. In the literature, three structures are given [12]: the relay control, the equivalent control method and the linear feedback with switched gains.

Originally, the most commonly used approach for control of electrical machines was the relay control since this one was more suitable for the structure of the power electronics converters.

With the advances made in power electronics and in data processing, the equivalent control and the linear feedback with switch gains approaches have substituted the relay control strategy.

Nevertheless, the robustness of the three SMC approaches is guaranteed only after the system reaches the switching surface and not necessary during the tracking phase. One solution to eliminate the sensitivity during the reaching phase consists in rotating or shifting the sliding surface in such a way that the existence of the sliding mode is guaranteed since the beginning of the motion [13].

2.2. Time-Varying Switching Surface

It is proposed in this part to shift the sliding line with a constant speed as proposed by [13] in order to have the sliding mode since the beginning of the motion.
For each kind of machines, the parameters describing the way the switching line moves will be defined in such a way that:
- the limitation of the current is satisfied:
  \[|i(t)| < I_{\text{max}}\] (maximum value supported by the armature)  
  \[1\]
- the integral of the absolute value of the motor shaft position error \(Q\) over the period of control action is minimized:
  \[Q = \int_{0}^{T} |X(t) - X_d| \, dt\]
  \[2\]
- the switching frequency is minimized and the delay due to the sampling period is compensated.

For this, the following time-varying switching line is introduced:
\[
S(X, \frac{dX}{dt}) = \frac{dX}{dt} + c \cdot X + \begin{cases} \alpha \cdot t + \beta & \text{for } t \leq T \\ \alpha \cdot T + \beta & \text{for } t > T \end{cases}
\]

(3)

where \(\alpha\), \(\beta\) and \(T\) are constant parameters chosen in such a way that the state of the motor belongs to the sliding line at time \(t=0\) and that the sliding line passes through the desired position \(X_d\) at time \(t=T\). Then, from these assumptions, the following relations can be deduced:
\[
\begin{align*}
\beta &= 0 \\
\alpha &= -cX_d / T
\end{align*}
\]

(4)

With the conventional SMC, the equation of the position is known once the trajectory reaches the sliding surface. With the proposed shifted switching surface, the position, the velocity and the acceleration can be expressed for any time by solving (3):

For \(t \leq T\):
\[
X = \frac{\alpha}{c^2} \cdot (\exp(-c^2t) - c^2t + 1)
\]
\[
\frac{dX}{dt} = \frac{\alpha}{c} \cdot (\exp(-c^2t) - 1)
\]
\[
\frac{d^2X}{dt^2} = -\alpha \cdot \exp(-c^2t)
\]

For \(t > T\):
\[
X = \frac{\alpha}{c^2} \cdot (\exp(-c^2T) / \alpha - 1) \cdot \exp(-c^2t) + X_d
\]
\[
\frac{dX}{dt} = -\frac{\alpha}{c} \cdot (\exp(-c^2X_d / \alpha - 1) - 1) \cdot \exp(-c^2t)
\]
\[
\frac{d^2X}{dt^2} = -\alpha \cdot (\exp(-c^2X_d / \alpha - 1) - 1) \cdot \exp(-c^2t)
\]

(5)

(6)

In the following parts, this time-varying switching surface will be associated to linear feedback with switched gains SMC for different kind of electrical machines (dc machine, three-phase induction machine and six-phase induction machine).

3. SMC APPLIED TO ELECTRICAL DRIVES

In this part, the time-varying switching surface associated to linear feedback with switched gains SMC as is applied to classical electrical machines such as dc machines and 3PIM. For this, a test bed that allows variations of load torque and inertia changes has been built up in the laboratory as shown on Fig 1. Dc machine, 3PIM pr 6PIM can be connected to this test bed.
3.1. Application to DC Motor Drive

In order to model the dynamics of the dc motor, the dc-dc converter and the mechanical load, a simplified model has been developed using the classical following equations:

\[
\begin{align*}
U & = R_i i + L_i \frac{di}{dt} + k_i \Omega \\
J \frac{d\Omega}{dt} & = k_i i - F \Omega - T_l \\
\Omega & = \frac{d\theta}{dt} \\
X & = k_i \theta = \frac{N_p}{2\pi}
\end{align*}
\]

where \(U\) and \(i\) are the armature voltage and current, \(R\) and \(L\) are the armature circuit resistance and self-inductance, \(k_e\) is the emf coefficient, \(k_c\) is the torque coefficient, \(\theta\) and \(\Omega\) are the motor shaft position and velocity, \(X\) is the position with respect to the number of pulses \(N_p\) of the encoder, \(J\) is the total moment of inertia of the drive, \(F\) is the viscous friction coefficient and \(T_l\) is the load torque. To define the topology of the proposed controller, the current controlled dc-dc converter will be considered as a positive constant gain \(k_a\) but the complete simulation will be performed using the model (7) without any simplification. Linear feedback with switched gains SMC is based on the modification of the linear feedback: a positive or a negative feedback is given according to the position of the representative working point in the phase plane as following:

\[
\begin{align*}
\text{If } S(X, \frac{dX}{dt}) < 0 \text{ then } H = -1 \text{ and } G = -1 \text{ (positive feedback)} \\
\text{If } S(X, \frac{dX}{dt}) > 0 \text{ then } H = 1 \text{ and } G = 1 \text{ (negative feedback)}
\end{align*}
\]

where \(S(X, \frac{dX}{dt}) = c \cdot \dot{X} + dX/dt = 0\) is the switching surface and \(\dot{X} = X - X_{ref}\).

Therefore, the control voltage \(u\) is composed of two terms \(u_1\) and \(u_2\) (see Fig. 2) related respectively to the feedback loop and to a feedforward part to remove the steady state error.
The sliding conditions are checked if the following condition is true:

\[ k_s \frac{J \dot{c}^2 - F \dot{c}'}{k_1 k_2 k_3} \]  

With \( c' = c \frac{1 + (A / c - 1) B}{1 - AT_c(A - c) \tau B} \)

\[ A = \frac{k_p k_s k_k k_d}{F}, \quad \tau = \frac{1}{F}, \quad B = 1 - \exp(-T_s / \tau) \]  

(11)

Where \( c' \) is the extreme switching line taking into account the sampling period \( T_s \) delay effect. Considering the mechanical switching line (7) and the current constraint (1), the following parameters can be obtained (where \( D \) is the maximum value for load torque) for the SMC with time varying switching surface:

\[ \alpha = -\frac{k_s k_k I - D}{J} \quad c = \frac{-2 \alpha}{X_s} \]  

(12)

Then, from the equations (7) and (12), the following parameters are obtained for the sliding surface:

\( \alpha = -71645 \quad c = 16.95 \quad k_p = 0.81 \)  

(13)

A low cost 16-bit microcontroller (Intel 80C196kb) has been chosen to implement the proposed control algorithm in order to suit with the small power drive application. This 16-bit microcontroller is cheaper than a DSP but it is suitable for electrical drive with adapted peripherals such as analog to digital converters, pulse width modulation (PWM) outputs, high speed input/output lines and other conventional input/output ports. The experimental position and trajectory in the phase plane when the jointed arm is first folded back \( (J = J_{\text{min}} = 0.740 \text{ mkg.m}^2) \) and then spread out \( (J = J_{\text{max}} = 1 \text{ mkg.m}^2) \) are represented on figure 3.a and 3.b.

The position for the two mechanical configurations \( J = J_{\text{min}} \) and \( J = J_{\text{max}} \) give two superposed trajectories. Even, in spite of the numerous commutations during the reaching phase, the trajectories in the phase plane are conservative and that is an experimental proof of robustness.

From the experimental results, it can be observed that the time-varying surface of the proposed SMC applied to the dc drive is insensitive to plant parameters changes since the beginning of the motion.
3.2. Application to Three-Phase Induction Machine

In this part, the same SMC strategy will be applied to a classical three-phase induction machine (3PIM). In order to modelize the dynamics of the 3PIM, the inverter and the mechanical load, a suitable model has been developed using a fourth-order state-space model for the electrical part and a second order for the mechanical part. For this, the stator and rotor variables are expressed in a rotating reference frame and it is supposed that the 0 component is removed for symmetry reason. Then the four equations for electrical variables are:

\[
\begin{align*}
V_{ds} &= R_s i_{ds} + \frac{d}{dt} \phi_{ds} - \omega_s \phi_{qs} \\
V_{qs} &= R_s i_{qs} + \frac{d}{dt} \phi_{qs} + \omega_s \phi_{ds} \\
V_{dr} &= R_r i_{dr} + \frac{d}{dt} \phi_{dr} = 0 \\
V_{qr} &= R_r i_{qr} + \frac{d}{dt} \phi_{qr} = 0
\end{align*}
\]

where:

\[
\begin{align*}
\phi_{ds} &= L_{m} i_{ds} + L_{m} i_{qs} \\
\phi_{qs} &= L_{m} i_{qs} + L_{m} i_{ds} \\
\phi_{dr} &= L_{m} i_{dr} + L_{m} i_{qr} \\
\phi_{qr} &= L_{m} i_{qr} + L_{m} i_{dr}
\end{align*}
\]

Then, the two electromechanical equations are:

\[
\begin{align*}
T_e &= \frac{3}{2} p (i_{ds} \phi_{ds} - i_{qs} \phi_{qs}) \\
J \frac{d}{dt} \omega &= T_e - F \omega - T_l
\end{align*}
\]

\(R_s\) and \(L_{ls}\) are the stator resistance and the stator leakage inductance, \(R_r\) and \(L_{lr}\) are the rotor resistance and the rotor leakage inductance, \(L_m\) is the mutual inductance, \(p\) is the number of pole pairs, \(T_e\) is electromagnetic torque, \(T_l\) is the load torque, \(J\) is the total inertia of the rotating part, \(F\) is the viscous friction coefficient, \(\omega\) and \(\theta\) are the speed and position of the rotor which are time variable, \(\omega_s\) is the pulsation of the stator currents, \(i_{ds}\) and \(i_{qs}\) (resp. \(i_{dr}\) and \(i_{qr}\)) are the direct and the quadratic components of the stator (resp. rotor) currents, \(V_{ds}\) and \(V_{qs}\) (resp. \(V_{dr}\) and \(V_{qr}\)) are the direct and the quadratic components of the stator (resp. rotor) voltages, \(\phi_{ds}\) and \(\phi_{qs}\) (resp. \(\phi_{dr}\) and \(\phi_{qr}\)) are the direct and the quadratic components of the stator (resp. rotor) fluxes.

Using indirect field oriented control (IFOC) concept, it can be noticed that the torque and the flux can be controlled separately using respectively the direct component \(i_{ds}\) and the quadratic component \(i_{qs}\) of the stator current. Then, the behavior of the 3PIM is given by the following equations:
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\[
\begin{align*}
V_a &= \sigma L_a \frac{di_a}{dt} + \left( R_a + R_L \frac{L_m^2}{L_T} \right) i_a - \sigma L_a \omega_L \varphi_a - \frac{L_m}{T_L} \varphi_r \\
V_p &= \sigma L_p \frac{di_p}{dt} + \left( R_p + R_L \frac{L_p^2}{L_T} \right) i_p + \sigma L_p \omega_L \varphi_p + \frac{L_m}{T_L} \omega \varphi_r \\
\end{align*}
\]

(16)

with

\[
\sigma = 1 - \frac{L^2}{L_L L_T}
\]

Then, on \( q \) axis, the transfer function in the Laplace domain of the rotor position can be expressed as following [7]:

\[
\frac{\theta(s)}{V_{qi}(s)} = \frac{L_{q}^2}{\sigma L_L L_S + R_L L_{s}^2 + R_s L_{m}^2} \frac{1}{(1 + T_s) L_s} \quad \frac{p L_m \varphi_s}{L_s} + \frac{1}{J_s^2 + F_s}
\]

(17)

By neglecting the electrical dynamics, which are faster than the mechanical ones, the transfer function (17) of the position becomes:

\[
\frac{\theta(s)}{V_{qi}(s)} = \frac{K_p K_c}{J_s^2 + F_s} \quad \text{with} \quad K_p = \frac{L_{q}^2}{R_L L_{s}^2 + R_s L_{m}^2} \quad \text{and} \quad K_c = \frac{p L_m \varphi_s}{L_s}
\]

(18)

Using the same approach on the \( d \)-axis, the transfer function of the rotor flux can be expressed:

\[
\varphi_r(s) = \frac{L_{d}^2}{\sigma L_L L_D + R_L L_{D}^2 + R_D L_{m}^2} \quad \frac{L_{m}}{(1 + T_D) L_D} \quad V_{di}(s)
\]

(19)

With \( V_{di} = V_a + \text{emf}_d \quad V_{qi} = V_p + \text{emf}_q \),

\[
\text{emf}_d = \omega_L \sigma L_L i_p + \frac{L_{m}}{L_D} R_d \varphi_r
\]

(20)

\[
\text{emf}_q = -\omega_L \sigma L_L i_a - \omega_L \frac{L_{m}}{L_D} \varphi_r + \frac{L_{m}}{L_D} \omega_L \varphi_r
\]

From the previous assumptions, a linear system with two single input single output equations is obtained (see fig. 4). Therefore the flux and torque regulations can be realised separately. The equations (18) and (19) are linear but an error or a variation of the machine parameters can break the decoupling. Therefore, it is suitable to use a robust control algorithm such as SMC.

![Figure 4 Position (a) and flux (b) control schemes](https://iaeme.com/Home/journal/IJEET)

The control algorithm has been implemented using a DSP board with the 50MHz floating point TMS320C31 DSP from Texas Instruments combined with the ADMC201 motion control coprocessor from Analog Devices. The voltage source inverter uses switches such as IGBT chosen to allow transient overload and sufficient switching frequencies of 20kHz. Fig. 5.a and
5b depict the experimental position evolution and the phase plane trajectories with the time-varying switching line as presented in previous part when $J=J_{\text{min}}$ and $J_{\text{max}}$. For this profile $\phi_r = 0.6 \, \text{Wb}$, $K_f = 100$, $\theta_{\text{ref}} = 400 \, \text{step}$, $K_p = 150$, $s_{1p} = -28$, $c' = 25.3$, $c = 25$. It can be noticed that, with the shifted switching surface, the trajectories in the phase plane (Fig. 5.b) are very close whatever the mechanical load and that the experimental position evolutions are superposed. From these experimental results, it can be seen that the robustness of the proposed approach between plant parameter variations is validated as for the dc machine.

Figure 5 Experimental position a) and phase plane trajectories b) for different inertias

3.3. Application to Six-Phase Induction Machine

As for a three-phase induction machine (3PIM), it is well known that the machine control algorithm has to be built in a dq0 rotating frame in order to generate and apply dc quantities in steady state for an efficient torque control. In this way, the 6PIM model has to be transformed into a dq0 rotating reference frame by using some basic assumptions. Hence, the windings have to be assumed sinusoidally distributed on the stator side, the rotor cage is equivalent to a six-phase wound rotor and the magnetic saturation and the core losses are neglected. Once these assumptions made, the electrical matrix equations in the stationary reference frame for the stator and the rotor can be written as following:

$$[V_s] = [R_s][I_s] + s([L_{ns}][I_s] + [L_{ns}][I_s])$$

(21)

$$0 = [R_s][I_s] + s([L_{ns}][I_s] + [L_{ns}][I_s])$$

(22)

where the current and the voltage quantities are:

$$[V_s] = \begin{bmatrix} v_a & v_c & v_b & v_c & v_b & v_c \end{bmatrix}^T$$

$$[I_s] = \begin{bmatrix} i_a & i_c & i_b & i_c & i_b & i_c \end{bmatrix}^T$$

$$[I_s] = \begin{bmatrix} i_a & i_c & i_b & i_c & i_b & i_c \end{bmatrix}^T$$

To control the 6PIM, the original model of the machine is decomposed in three two-dimensional orthogonal subspaces $\alpha\beta$, $z_1z_2$, $\alpha_0\alpha_2$ by using the transformation matrices $T_6$ and $T_2$ [9]. Then the control scheme is the following:
The control programs are developed under MATLAB/Simulink® on a personal computer PC1 and compiled in the personal computer PC2. PC1 and PC2 are connected via an Ethernet link. A FPGA card connected with PC2 data bus has been used to generate the control signals for the 12 VSI switches. The six stator phase currents are measured using shunt-based current sensors of 20kHz frequency bandwidth connected to a 12-bit data acquisition system which is integrated in the VSI block.

Fig. 7.a and 7.b depict respectively the position response and the phase plane trajectories in healthy mode for $J_{min} = 9.5 \times 10^{-5}$ kg.m$^2$ and $J_{max} = 26.12 \times 10^{-5}$ kg.m$^2$ with the time-varying switching surface while Fig. 7.c and 7.d represent the same value when two consecutive stator phases are missing. It can be seen that with the time-varying switching surface, the responses are insensitive to both inertia changes and also faulted conditions since the beginning of the motion.

**Figure 6** SMC applied to position control of 6PIM

**Figure 7** SMC responses with time varying surface in healthy a) b) mode and with two open phases c) d)
4. CONCLUSION
A structure of SMC based on translation of the sliding surface since the beginning the motion has been presented for position control of electrical drives. The switching line has been moved by using a constant speed in such a way that the physical limitation of the stator current can be satisfied as well as the integral of the absolute value of the shaft position is minimized and that the switching frequency is also minimized around the final switching line. This control strategy has been applied to position control of both dc machine, 3PIM and 6PIM. The experimental results have shown that by using the proposed technique, the sliding mode occurs since the very beginning of the motion and the behaviour of the control system is fully insensitive to both mechanical configuration changes and faulted mode conditions (for 6PIM). This algorithm can be a solution for applications such as systems used in medicine, nuclear or aerospace for which a robust position control is required whatever the mechanical configuration is and whenever a faulted operation mode can occur.

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